MULTIMEDIA UNIVERSITY

STUDENT IDENTIFICATION NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2015/2016 SESSION

BQT1614 – QUANTITATIVE ANALYSIS

(All Sections / Groups)

5th OCTOBER 2015
9:00 a.m. – 12:00 noon
(3 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of 9 pages **excluding** the cover page.
2. This question paper consists of 5 structured questions. Attempt **ALL** questions.
3. Students are allowed to use non-programmable scientific calculators with no restrictions.
4. Statistical tables are attached at the end of the question paper.
5. Please use **pen** to write the answers.
6. Please print all your answers in the **Answer Booklet** provided.

Question 1

(a) A consumer agency surveyed all 2500 families living in a small town to collect data on the number of cars owned by them. The following table lists the frequency distribution of the data collected by this agency.

Number of cars owned	0	1	2	3	4
Number of families	120	970	730	410	270

i) Construct the probability distribution table for the number of cars owned by these families. [2 marks]

ii) Determine the expected number of cars owned and its variance. [5 marks]

iii) Calculate $E(4X^2 - 5X) + Var(2X)$. [2 marks]

(b) A company has 2 machines that produce toys. Machine A produces 23% defective toys, while machine B produces only 8% defective toys. In addition, the machine B produces 3 times as many toys as machine A does.

i) Draw the tree diagram. [3 marks]

ii) What is the probability that a randomly chosen toy produced by the company is defective? [4 marks]

iii) Given a randomly chosen toy was tested and found to be non-defective, what is the probability it was produced by machine B? [4 marks]

[Total 20 Marks]

Question 2

(a) Let X denote the time in hours needed to locate and correct a problem in the software that governs the timing of traffic lights in the downtown area of a large city. Assume that X is normally distributed with mean 10 hours and variance of 9 hours.

i) Find the probability that the next problem will require at most 15 hours to locate and correct the problem. [2 marks]

ii) Find the probability that the next problem will require between 7 hours and 8.5 hours to locate and correct the problem. [5 marks]

iii) The fastest 5% of repairs take at most how many hours to complete? [4 marks]

(b) An average of 4.8 students drop by FOM office to meet with the Deputy Dean of Student and Staff Affairs for enquiries per week (weekdays).

i) Find the probability that 3 or 4 students drop by during first week of the trimester. [4 marks]

ii) Find the probability that at most 2 students drop by on Wednesday. [5 marks]

[Total = 20 Marks]

Continued.....

Question 3

(a) Pond's Age-Defying Complex, a cream with alpha-hydroxy acid, advertises that it can reduce wrinkles and improve the skin. In a study published in *Archives of Dermatology*, 33 women over age 40 used a cream with alpha-hydroxy acid for twenty-two weeks. At the end of the study period, 23 of the women exhibited skin improvement (as judged by a dermatologist). Test whether there is evidence that the cream will improve the skin of more than 60% of women of over age 40. (Use $\alpha = 0.05$) [8 marks]

(b) Media observers have been examining the number of minutes devoted to business and financial news during the half-hour evening news broadcasts of two local television channels. For each channel, they have randomly selected 10 weekday broadcasts and observed the number of minutes spent on business and financial news during that broadcast. The times measured in these independent samples are shown below.

Channel A: 3.8, 2.7, 4.9, 3.4, 3.7, 4.5, 4.2, 2.8, 3.5, 4.6 minutes
Channel B: 3.6, 4.0, 4.5, 5.2, 4.8, 4.3, 5.7, 3.5, 3.7, 5.8 minutes

Assuming normal populations with equal standard deviations test at 0.05 level of significance whether there is any difference in the mean time spent on business and financial news for the two channels. [12 marks]

[Total = 20 Marks]

Question 4

A corporation administers an aptitude test to all new sales representatives. Management is interested in the extent to which this test is able to predict their eventual success. The accompanying table records average weekly sales (in thousands of dollars) and aptitude test scores for a random sample of twelve representatives.

Weekly Sales	10	12	28	24	18	16	15	12	14	22	13	26
Test Score	55	60	85	75	80	85	65	60	70	72	65	82

(a) What are the independent and dependent variables? [2 marks]
(b) Using the correlation coefficient, identify the type of relationship that exists between the variables. [5 marks]
(c) Compute the coefficient of determination and interpret its meaning. [3 marks]
(d) Determine the least squares linear regression equation to predict the weekly sales. [5 marks]

Continued.....

(e) Interpret the meaning of the slope coefficient in this problem. [2 marks]

(f) Estimate the weekly sales of the sales representative whose test score is 83 and comment on the reliability of the estimate. [3 marks]

[Total = 20 Marks]

Question 5

(a) Starting on her 35th birthday, a woman saves $200 at the end of each month with a view to retiring on her 50th birthday.

i) What is the value of her savings if the interest rate is 8% compounded monthly? [4 marks]

ii) From the age of 50, how much should she withdraw each month if the fund is to last for the next 15 years, assuming the interest rate is 9% compounded monthly? [4 marks]

(b) How long should it take for an investment of RM3700 to grow to RM5,000 when interest is compounded annually at 6.5%? [3 marks]

(c) Find the effective rate of interest corresponding to a nominal rate of 4.5% per year compounded quarterly. [3 marks]

(d) The hotel industry is very interested in understanding how tourists spend money. In order to measure the price and demand changes in two important components of a tourist's budget, a statistician computed the average cost and demand of a hotel room (one night), a meal in 2000 and in 2008. The results of these computations are shown in the accompanying table.

Item	Year 2000		Year 2008	
	Price	Qty	Price	Qty
Hotel (night)	85	4	120	4
meal	13	8	20	6

i) Compute Laspeyres price index for 2008 using 2000 as the base period. [3 marks]

ii) Compute Paasche price index for 2008 using 2000 as the base period. [3 marks]

[Total = 20 Marks]

End of Question

STATISTICAL FORMULAE

A. DESCRIPTIVE STATISTICS

Mean $(\bar{x}) = \dfrac{\sum_{i=1}^{n} X_i}{n}$

Standard Deviation $(s) = \sqrt{\dfrac{\sum_{i=1}^{n} X_i^2}{n-1} - \dfrac{(\sum_{i=1}^{n} X_i)^2}{n(n-1)}}$

Coefficient of Variation $(CV) = \dfrac{\sigma}{\bar{X}} \times 100$

Pearson's Coefficient of Skewness $(S_k) = \dfrac{3(\bar{X} - Median)}{s}$

B. PROBABILITY

P (A or B) = P (A) + P (B) – P (A and B)

P (A and B) = P (A)× P (B) if A and B are independent

P (A | B) = P (A and B) ÷ P (B)

Poisson Probability Distribution

If X follows a Poisson Distribution , $P(\lambda)$ where $P(X = x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$

then the mean = $E(X) = \lambda$ and variance = $VAR(X) = \lambda$

Binomial Probability Distribution

If X follows a Binomial Distribution B(n, p) where $P(X = x) = {}^nC_x p^x q^{n-x}$

then the mean = $E(X) = np$ and variance = $VAR(X) = npq$ where $q = 1 - p$

Normal Distribution

If X follows a Normal distribution, $N(\mu, \sigma)$ where $E(X) = \mu$ and $VAR(X) = \sigma^2$

then $Z = \dfrac{X - \mu}{\sigma}$

C. CONFIDENCE INTERVAL ESTIMATION AND SAMPLE SIZE DETERMINATION

(100 – α) % Confidence Interval for Population Mean (σ Known) = $\mu = \overline{X} \pm Z_{\alpha/2}\left(\sigma/\sqrt{n}\right)$
(100 – α)% Confidence Interval for Population Mean (σ Unknown) = $\mu = \overline{X} \pm t_{\alpha/2,n-1}\left(s/\sqrt{n}\right)$
(100 – α)% Confidence Interval for Population Proportion = $\pi = p \pm Z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}}$
Sample Size Determination for Population Mean $= n \geq \left[\frac{(Z_{\alpha/2})\sigma}{E}\right]^2$
Sample Size Determination for Population Proportion = $n \geq \frac{Z^2 \times p(1-p)}{E^2}$
Where E = Limit of Error in Estimation

D. HYPOTHESIS TESTING

One Sample Mean Test	
Standard Deviation (σ) Known	**Standard Deviation (σ) Not Known**
$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$	$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$
One Sample Proportion Test	
$Z = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}}$	
Two Sample Mean Test	
Standard Deviation (σ) Known	**Standard Deviation (σ) Not Known**
$z = \frac{(\overline{x_1} - \overline{x_2}) - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}}$	$t = \frac{(\overline{x_1} - \overline{x_2}) - (\mu_1 - \mu_2)}{\sqrt{S_p^2\left(1/n_1 + 1/n_2\right)}}$ where $S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{(n_1+n_2-2)}$
Two Sample Proportion Test	
$Z = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$ where $\bar{p} = \frac{X_1 + X_2}{n_1 + n_2}$; $p_1 = \frac{X_1}{n_1}$; $p_2 = \frac{X_2}{n_2}$	
where X_1 and X_2 are the number of successes from each population	

E. REGRESSION ANALYSIS

Simple Linear Regression

$Y_i = \beta_0 + \beta_1 X_1 + \varepsilon_I$ Where $\beta_0 = \bar{Y} - \beta_1 \bar{X}$ and $\beta_1 = \dfrac{\sum XY - \left[\dfrac{\sum X \sum Y}{n}\right]}{\left[\sum X^2 - \left(\dfrac{(\sum X)^2}{n}\right)\right]}$

Correlation Coefficient

$$r = \frac{\sum XY - \left[\dfrac{\sum X \sum Y}{n}\right]}{\sqrt{\left[\sum X^2 - \left(\dfrac{(\sum X)^2}{n}\right)\right]\left[\sum Y^2 - \left(\dfrac{(\sum Y)^2}{n}\right)\right]}} = \frac{COV(X,Y)}{\sigma_X \sigma_Y}$$

ANOVA Table for Regression

Source	Degrees of Freedom	Sum of Squares	Mean Squares
Regression	*1*	SSR	MSR = SSR/1
Error/Residual	$n-2$	SSE	MSE = SSE/$(n-2)$
Total	$n-1$	SST	

Test Statistic for Significance of the Predictor Variable

$t_i = \dfrac{b_i}{S_{b_i}}$ and the critical value $= \pm t_{\alpha/2,(n-p-1)}$

Where p = number of predictor

F. INDEX NUMBERS

Simple Price Index $P = \dfrac{p_t}{p_0} \times 100$	**Laspeyres Quantity Index** $P = \dfrac{\sum p_0 q_t}{\sum p_0 q_0} \times 100$
Aggregate Price Index $P = \dfrac{\sum p_t}{\sum p_0}(100)$	**Paasche Quantity Index** $P = \dfrac{\sum p_t q_t}{\sum p_t q_0} \times 100$
Laspeyres Price Index $P = \dfrac{\sum p_t q_0}{\sum p_0 q_0} \times 100$	**Fisher's Ideal Price Index** $\sqrt{(Laspeyres\ Price\ Index)(Paasche\ Price\ Index)}$
Paasche Price Index $P = \dfrac{\sum p_t q_t}{\sum p_0 q_t} \times 100$	**Value Index** $V = \dfrac{\sum p_t q_t}{\sum p_0 q_0} \times 100$

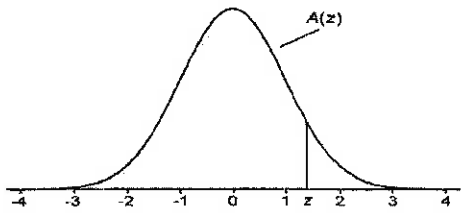
G. FINANCIAL MATHEMATICS

Simple Interest	**Compounded Interest**
$I = Pnr$ $A = P(1+nr)$	$I_t = P[(1+i)^n - 1]$ $A_t = P(1+i)^n$
Effective Rate of Interest $r_{\text{effective}} = \left(1+\frac{r}{m}\right)^m - 1$	**Sinking Fund** $S = R\left[\frac{(1+i)^n - 1}{i}\right]$
Future Value $A_t = P(1+i)^n$	**Present Value** $P = A(1+i)^{-n}$
Future Value of an Annuity $FV_A = R\left[\frac{(1+i)^n - 1}{i}\right]$	**Present Value of an Annuity** $PV_A = R\left[\frac{1-(1+i)^{-n}}{i}\right]$
Amortization $R = \left[\frac{P(i)}{1-(1+i)^{-n}}\right]$	

STATISTICAL TABLES 1

TABLE A.1

Cumulative Standardized Normal Distribution



$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:

z	$A(z)$	
1.645	0.9500	Lower limit of right 5% tail
1.960	0.9750	Lower limit of right 2.5% tail
2.326	0.9900	Lower limit of right 1% tail
2.576	0.9950	Lower limit of right 0.5% tail
3.090	0.9990	Lower limit of right 0.1% tail
3.291	0.9995	Lower limit of right 0.05% tail

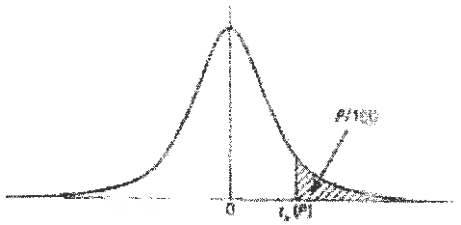
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999							

TABLE 10. PERCENTAGE POINTS OF THE t-DISTRIBUTION

This table gives percentage points $t_\nu(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{\nu\pi}} \frac{\Gamma(\frac{1}{2}\nu + \frac{1}{2})}{\Gamma(\frac{1}{2}\nu)} \int_{t_\nu(P)}^{\infty} \frac{dt}{(1 + t^2/\nu)^{\frac{1}{2}(\nu+1)}}.$$

Let X_1 and X_2 be independent random variables having a normal distribution with zero mean and unit variance and a χ^2-distribution with ν degrees of freedom respectively; then $t = X_1/\sqrt{X_2/\nu}$ has Student's t-distribution with ν degrees of freedom, and the probability that $t \geqslant t_\nu(P)$ is $P/100$. The lower percentage points are given by symmetry as $-t_\nu(P)$, and the probability that $|t| \geqslant t_\nu(P)$ is $2P/100$.



The limiting distribution of t as ν tends to infinity is the normal distribution with zero mean and unit variance. When ν is large interpolation in ν should be harmonic.

P	40	30	25	20	15	10	5	2·5	1	0·5	0·1	0·05
$\nu = 1$	0·3249	0·7265	1·0000	1·3764	1·963	3·078	6·314	12·71	31·82	63·66	318·3	636·6
2	0·2887	0·6172	0·8165	1·0607	1·386	1·886	2·920	4·303	6·965	9·925	22·33	31·60
3	0·2767	0·5844	0·7649	0·9785	1·250	1·638	2·353	3·182	4·541	5·841	10·21	12·92
4	0·2707	0·5686	0·7407	0·9410	1·190	1·533	2·132	2·776	3·747	4·604	7·173	8·610
5	0·2672	0·5594	0·7267	0·9195	1·156	1·476	2·015	2·571	3·365	4·032	5·893	6·869
6	0·2648	0·5534	0·7176	0·9057	1·134	1·440	1·943	2·447	3·143	3·707	5·208	5·959
7	0·2632	0·5491	0·7111	0·8960	1·119	1·415	1·895	2·365	2·998	3·499	4·785	5·408
8	0·2619	0·5459	0·7064	0·8889	1·108	1·397	1·860	2·306	2·896	3·355	4·501	5·041
9	0·2610	0·5435	0·7027	0·8834	1·100	1·383	1·833	2·262	2·821	3·250	4·297	4·781
10	0·2602	0·5415	0·6998	0·8791	1·093	1·372	1·812	2·228	2·764	3·169	4·144	4·587
11	0·2596	0·5399	0·6974	0·8755	1·088	1·363	1·796	2·201	2·718	3·106	4·025	4·437
12	0·2590	0·5386	0·6955	0·8726	1·083	1·356	1·782	2·179	2·681	3·055	3·930	4·318
13	0·2586	0·5375	0·6938	0·8702	1·079	1·350	1·771	2·160	2·650	3·012	3·852	4·221
14	0·2582	0·5366	0·6924	0·8681	1·076	1·345	1·761	2·145	2·624	2·977	3·787	4·140
15	0·2579	0·5357	0·6912	0·8662	1·074	1·341	1·753	2·131	2·602	2·947	3·733	4·073
16	0·2576	0·5350	0·6901	0·8647	1·071	1·337	1·746	2·120	2·583	2·921	3·686	4·015
17	0·2573	0·5344	0·6892	0·8633	1·069	1·333	1·740	2·110	2·567	2·898	3·646	3·965
18	0·2571	0·5338	0·6884	0·8620	1·067	1·330	1·734	2·101	2·552	2·878	3·610	3·922
19	0·2569	0·5333	0·6876	0·8610	1·066	1·328	1·729	2·093	2·539	2·861	3·579	3·883
20	0·2567	0·5329	0·6870	0·8600	1·064	1·325	1·725	2·086	2·528	2·845	3·552	3·850
21	0·2566	0·5325	0·6864	0·8591	1·063	1·323	1·721	2·080	2·518	2·831	3·527	3·819
22	0·2564	0·5321	0·6858	0·8583	1·061	1·321	1·717	2·074	2·508	2·819	3·505	3·792
23	0·2563	0·5317	0·6853	0·8575	1·060	1·319	1·714	2·069	2·500	2·807	3·485	3·768
24	0·2562	0·5314	0·6848	0·8569	1·059	1·318	1·711	2·064	2·492	2·797	3·467	3·745
25	0·2561	0·5312	0·6844	0·8562	1·058	1·316	1·708	2·060	2·485	2·787	3·450	3·725
26	0·2560	0·5309	0·6840	0·8557	1·058	1·315	1·706	2·056	2·479	2·779	3·435	3·707
27	0·2559	0·5306	0·6837	0·8551	1·057	1·314	1·703	2·052	2·473	2·771	3·421	3·690
28	0·2558	0·5304	0·6834	0·8546	1·056	1·313	1·701	2·048	2·467	2·763	3·408	3·674
29	0·2557	0·5302	0·6830	0·8542	1·055	1·311	1·699	2·045	2·462	2·756	3·396	3·659
30	0·2556	0·5300	0·6828	0·8538	1·055	1·310	1·697	2·042	2·457	2·750	3·385	3·646
32	0·2555	0·5297	0·6822	0·8530	1·054	1·309	1·694	2·037	2·449	2·738	3·365	3·622
34	0·2553	0·5294	0·6818	0·8523	1·052	1·307	1·691	2·032	2·441	2·728	3·348	3·601
36	0·2552	0·5291	0·6814	0·8517	1·052	1·306	1·688	2·028	2·434	2·719	3·333	3·582
38	0·2551	0·5288	0·6810	0·8512	1·051	1·304	1·686	2·024	2·429	2·712	3·319	3·566
40	0·2550	0·5286	0·6807	0·8507	1·050	1·303	1·684	2·021	2·423	2·704	3·307	3·551
50	0·2547	0·5278	0·6794	0·8489	1·047	1·299	1·676	2·009	2·403	2·678	3·261	3·496
60	0·2545	0·5272	0·6786	0·8477	1·045	1·296	1·671	2·000	2·390	2·660	3·232	3·460
120	0·2539	0·5258	0·6765	0·8446	1·041	1·289	1·658	1·980	2·358	2·617	3·160	3·373
∞	0·2533	0·5244	0·6745	0·8416	1·036	1·282	1·645	1·960	2·326	2·576	3·090	3·291